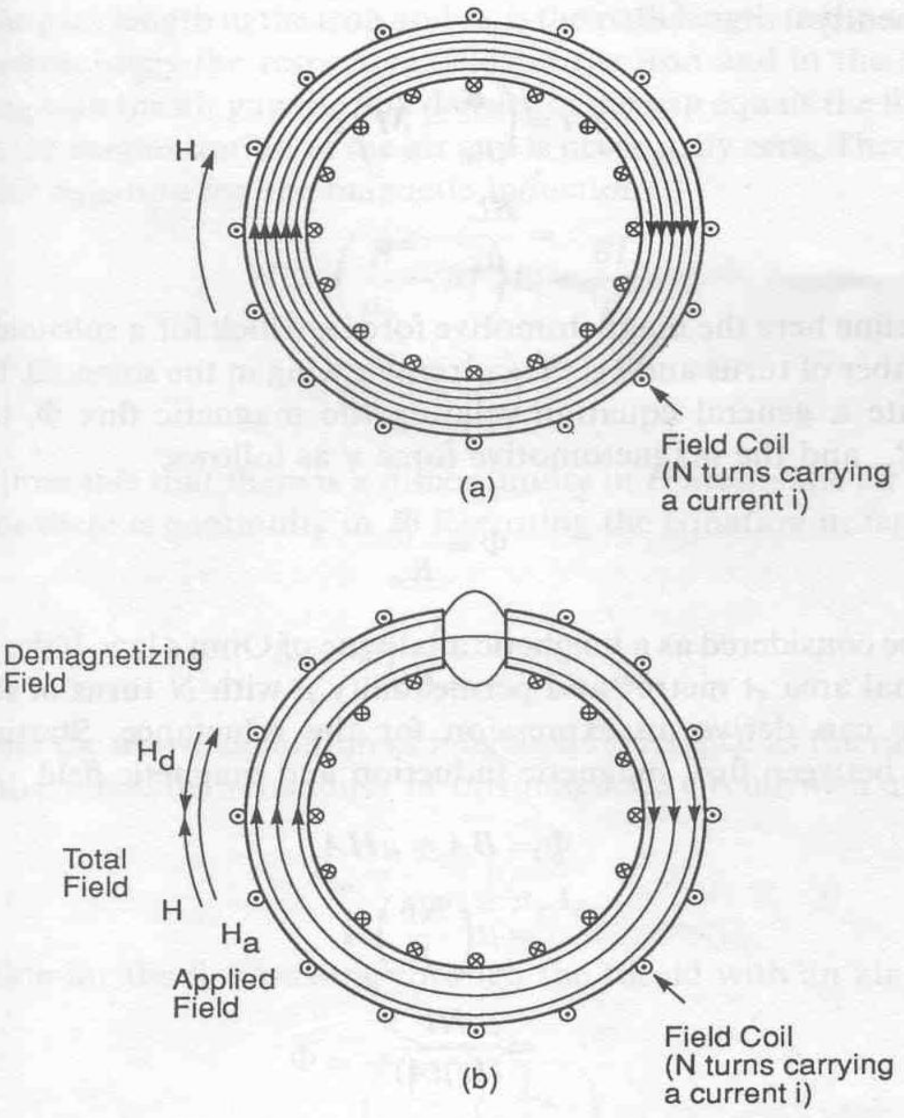
Magnetic circuit and reluctance

(How do we calculate magnetic field or flux in situations where we have an air gap or two materials with diferrent magnetic properties?)



Magnetic Circuits: (a) closed, and (b) open.

Fig. 2.6 Magnetic flux path in (a) an iron toroid forming a closed magnetic path, and (b) an iron toroid with an air gap. The flux lines are shown as concentric circles within the iron. The air gap increases the reluctance of the magnetic circuit and reduces the flux density in the iron as well as in the gap.

1. A magnetic flux path is interrupted by an air gap are of practical importance.
2. The problems encountered here are more complicated than in calculating the flux in a single material.
3. The magnet engineer is often of calculating the magnetic flux in magnetic circuits with a combination of an iron and air core

(i) In closed circuit case

(a ring of iron is wound with *N* turns of a solenoid which carries a current *i*)

Magnetic field → $H = \frac{Ni}{L}$ $\begin{pmatrix} N: \text{Turns} \\ L: \text{The average length of the ring} \end{pmatrix}$

Flux density passing → in the ring

$$B = \mu_o(H + M)$$

$$B = \mu_o\left(\frac{Ni}{L} + M\right)$$

$$\Rightarrow Ni = \left(\frac{B}{\mu_o} - M\right) L = \frac{B\,L}{\mu}$$

Ampere's law → $Ni = \int_{\substack{\text{circuit}\\\text{path}}} H\,dl$

$$Ni = HL \quad (B = \mu H)$$

$$Ni = \frac{B}{\mu} L$$

We can define here **the magnetomotive force *η*** which for a solenoid is *Ni*

(*N*: the number of turns
i: the current flowing in the solenoid)

(A magnetic analogue of flow Ohm's law)

(We can formulate a general equation relating the magnetic flux)

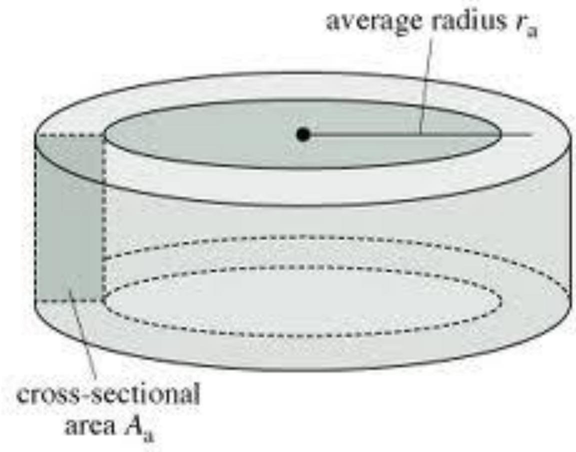
$$\Phi = \frac{\eta}{R_m} \qquad \Phi : \text{magnetic flux}$$

$$\Updownarrow \qquad R_m : \text{magnetic relutcance}$$

$$i = \frac{V}{R} \qquad \Phi \Leftrightarrow i$$

$$\eta \Leftrightarrow V$$

$$R_m \Leftrightarrow R$$



If the iron ring has

cross-section area A(m²)
permeability μ
Turns of solenoid N
length L

Starting from the relationship between flux, magnetic induction and magnetic field

$$\Phi = B \cdot A = \mu H \cdot A$$
$$= \mu \left(\frac{Ni}{L} \right) A$$
$$= \frac{Ni}{\left(L \big/ \mu A \right)}$$

- The term $L/\mu A$ is the magnetic reluctance of path.

- Magnetic reluctance is series in a magnetic circuit may be added

is analogous to

$$R_m = \frac{L}{\mu A} \Leftrightarrow R = \rho \frac{\ell}{A}$$

$$\rho \Leftrightarrow \frac{1}{\mu}$$

$$\sigma \Leftrightarrow \mu$$

Magnetic circuit	Electrical circuit
$\text{Flux }(\Phi) = \dfrac{\text{magnetomotive force}}{\text{reluctance}}$	$\text{Current }(i) = \dfrac{\text{electromotive force}}{\text{resistance}}$
$\text{Reluctance} = \dfrac{l}{\mu A}$	$\text{Resistance} = R = \rho \dfrac{l}{A}$
$\text{Reluctivity} = \dfrac{1}{\mu}$	$\text{Resistivity} = \rho$
$\text{permeability} = \mu$	$\text{conductivity} = \sigma = \dfrac{1}{\rho}$

(ii) In open circuit case

- If the air gap is small, there will be little leakage of the flux at the gap, but $B = \mu H$ can no longer apply since the μ of air and the iron ring are different. (Ignoring demagnetizing effects for the present calculation)

From Ampere's law

There is a discontinuity in H across the air gap, there is continuity in B

$$Ni = \int_{\text{closedpath}} H dl = H_i L_i + H_a L_a$$

L_i: the path length in the iron
L_a: the path length in the air
H_i: field in the iron
H_a: field in the air

$$Ni = \left(\frac{B}{\mu_0} - M\right) L_i + \frac{B L_a}{\mu_a} = B\left(\frac{L_i}{\mu_i} + \frac{L_a}{\mu_a}\right)$$

For the ring with the air gap, B in the gap equals B in the iron, but M in the air gap is necessarily zero.

$$\Phi = B \cdot A$$

$$B = \frac{\Phi}{A}$$

Rewriting the eq in terms of flux Φ

$$N_i = \Phi\left(\frac{L_i}{A_i \mu_i} + \frac{L_a}{A_a \mu_a}\right)$$

$$Ni = \Phi R_m$$
$$\downarrow$$
$$\eta$$

$$\mathrm{R}_m = \frac{L_i}{A_i \mu_i} + \frac{L_a}{A_a \mu_a}$$

$$\Phi = \frac{Ni}{\left(\dfrac{L_i}{\mu_i A_i} + \dfrac{L_a}{\mu_a A_a}\right)}$$

$$\left(\Phi = \frac{\eta}{R_m}\right)$$

(When the air gap is introduced) Φ in the ring is reduced, because it requires more energy to drive the same flux across the air gap than through an equal volume of the iron due to the much lower permeability of the air.

Magnetic Measurement

- There are several methods available and these divide broadly into two categories of field measurement: those which depend on:
  - ① magnetic induction using coils
  - ② measuring changes in various properties of materials caused by the presence of a magnetic field.

- The measurements of magnetization
  - ① force measurements such as in the torque magnetometer.
  - ② gradiometer measurement which measure the difference in magnetic induction with and without the sample present.

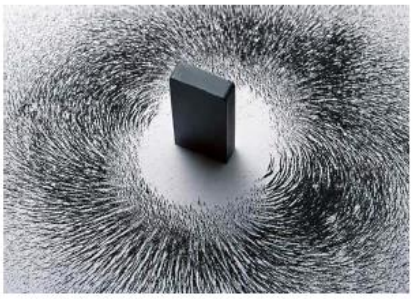
- Induction methods
  - ① stationary – coil method
  - ② moving – coil method
  - ③ rotating – coil method
  - ④ vibrating – coil magnetometer
  - ⑤ vibrating – sample magnetometer(VSM)

磁場的來源

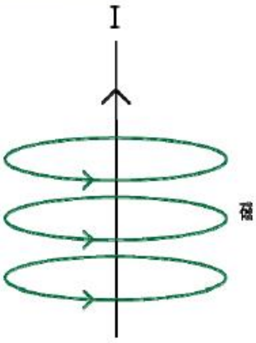
磁性量測

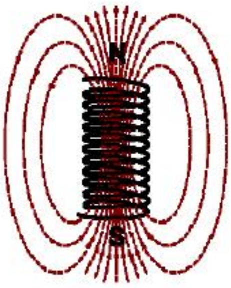
直流磁場

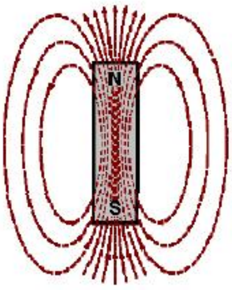
1. 永磁材料：磁鐵-鐵氧體、釹鐵硼



2. 恆定電流源







- Induction method

(How can the strength of an external field be measured from the e.m.f. generated in an electrical circuit due to a change in flux linking the circuit?)

- The induction method of measuring magnetic fluxs are all dependent on Faraday’s law of electromagnetic induction.

$$V = -N\frac{d\phi}{dt}$$

(The e.m.f. induces in a circuit is equal to the rate of change of flux linking the circuit.)

$$V = -NA\frac{dB}{dt}$$

A: the cross-sectional area the coil
N: the number of turns

$$B = \frac{\Phi}{A} \qquad V = -\mu_0 NA\frac{dH}{dt}$$

In free space
$B = \mu_0 H$

The induced voltage is increased if B is increased while H is maintained constant by inserting a high-permeability core into the coil.

① stationary – coil method

(How can the rate of change of field be found using the e.m.f. generated in a stationary coil?)

- A stationary – coil method can only measure the rate of change of magnetic induction by measuring the induced voltage.

$$V = -NA\frac{dB}{dt}$$

$$B = -\frac{1}{NA}\int v\,dt$$

② Moving – coil(extraction) method

(How is the magnetic induction measured when a search coil is placed in a magnetic field and rapidly removed ?)

$$V = -NA\frac{dB}{dt}$$

$$\int v\,dt = -NA(B_f - B_i)$$

B_i: the initial magnetic induction

B_f: the final magnetic induction

- The angular deflection ϕ is proportional to $\int v\,dt$

$$\phi = \text{constant} \times \int v\,dt$$

$$= \text{constant} \times NA(B_f - B_i)$$

- The deflection of the galvanometer is therefore proportional to the change in magnetic induction.

③ Rotating-coil method

(Can we make use of a rotating coil to generate the necessary induced e.m.f. in a static field ?)

- In order to obtain a measurement of the magnetic induction it is also possible to use various moving-coil instruments. The simplest of these is the rotating coil which rotates at a fixed angular velocity ω.

$$B(t) = B\cos\omega t$$

$$\begin{aligned} V &= -NA\frac{dB}{dt} \\ &= -\mu_0 NA\frac{dH}{dt} \\ &= -\mu_0 NA\omega H\sin\omega t \end{aligned}$$

- The amplitude of the voltage generated by the rotating coil is proportional to the magnetic induction and therefore the amplitude can be used to measure B or H in free space.

④ Vibrating – coil magnetometer

(How can the linear displacement of a coil in a magnetic field be used to measure the strength of the field ?)

The vibrating – coil magnetometer is based on the same principles as the previous technique, but is used primarily as a method of determining the magnetization M.

$$B_m = \mu_0(H + M)$$

↑

B surrounding the sample

$$B_0 = \mu_0 H$$

↑

B linking the coil when it has moved away from the sample

The change in induction is then simply $\Delta B = \mu_0 M$

The flux change caused when coil is removed from the specimen

$$\int v dt = -NA\mu_0 M$$

The output of the vibrating coil magnetometer is independent of H, but is independent on M.

⑤ Vibrating – sample magnetometer (VSM)

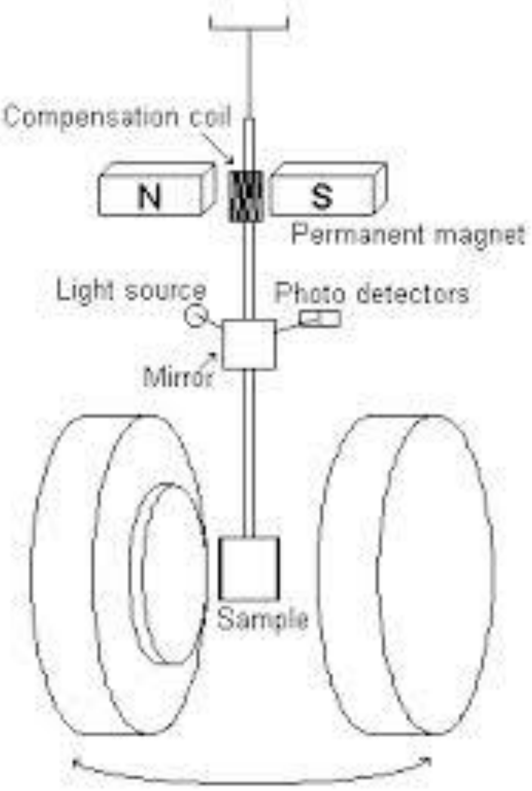
(If the specimen is moved instead of the coil how can the induced voltage be used to determine the magnetization of the specimen ?)

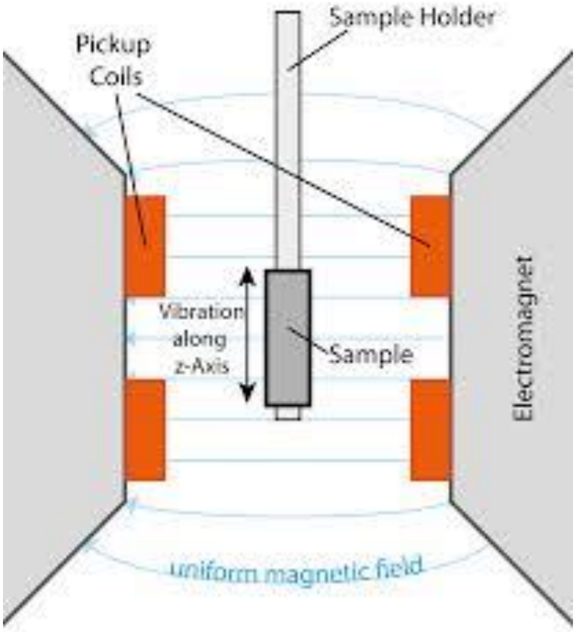
- The VSM is similar in principle to the vibrating-coil magnetometer but differs in so far as the sample is moved instead of the coil.

- A VSM is a really a gradiometer measuring the difference in magnetic induction between a region of space with and without the specimen. It therefore gives a direct measure of the magnetization (M

VSM


Model 7407
LakeShore


Compensation coil
N
S
Permanent magnet
Light source
Photo detectors
Mirror
Sample


Pickup
Coils
Sample Holder
Vibration
along
z-Axis
Sample
Electromagnet
uniform magnetic field

Methods depending on change in material properties

① Hall effect magnetometers
② Magnetoresistors
③ Magnetostrictive devices
④ Magneto-optic method
⑤ Thin-film magnetometers

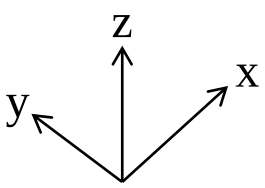
(How can magnetic field strengths be determined from changes in material properties ?)

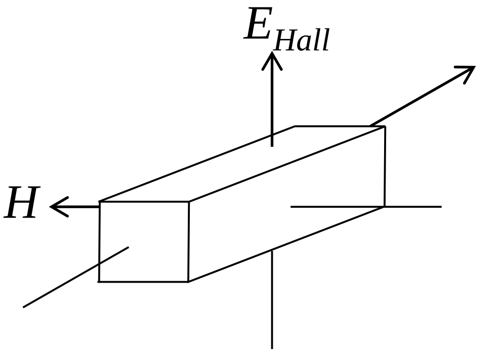
①Hall effect magnetometers

(How does the presence of a magnetic field alter the motion of charge carries ?)

- When a magnetic field is applied to a conducting material carrying an electric current, there is a transverse Lorentz force on the charge carries given by $\vec{F} = \mu_0 e v \times \vec{H}$

e: a single charge
v: velocity
H: field of strength



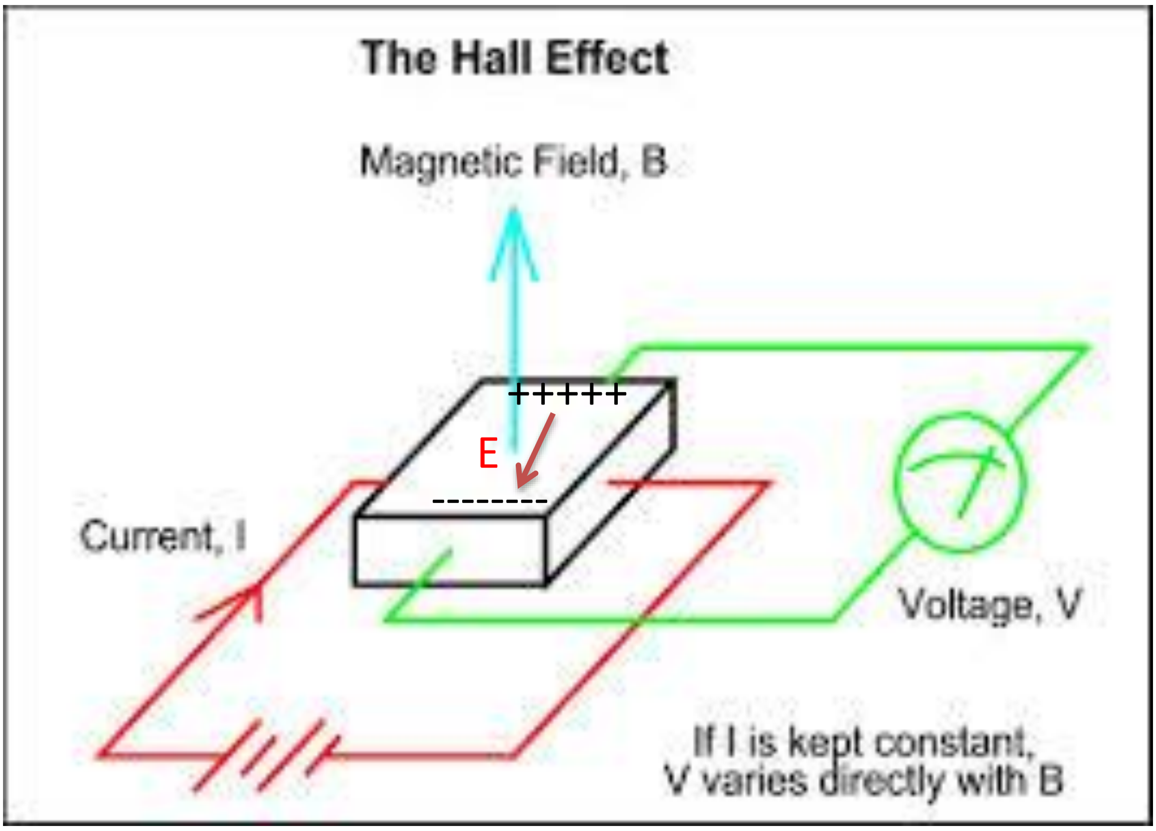


$\vec{F} = e\vec{E}_{Hall}$

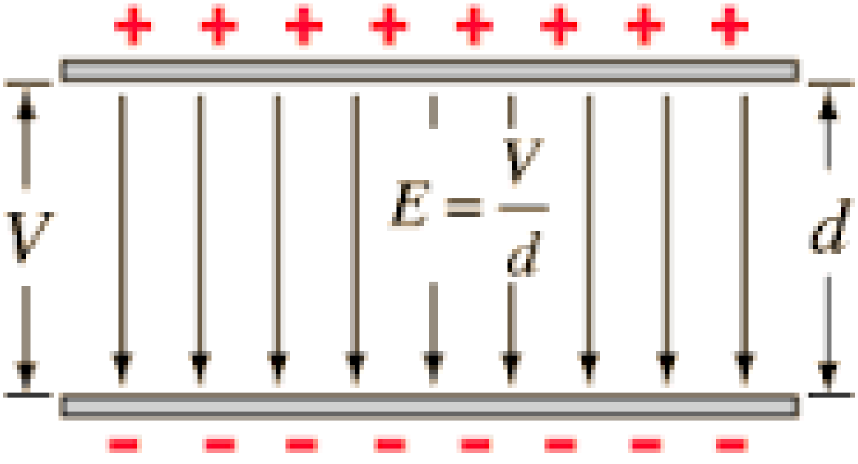
(If the electric current passes in the χ direction and the magnetic field passes in the y direction of a slab of semiconductor of dimension ℓ_x ℓ_y ℓ_z ,the Hall e.m.f. will be along the z axis.)

where E is the electric field we can consider that the force is due to an equivalent electric field E_{hall}, know as the Hall-field)

$$\vec{E}_{Hall} = \mu_0 v \times \vec{H}$$


The Hall Effect
Magnetic Field, B
++++++
E

Current, I
Voltage, V
If I is kept constant,
V varies directly with B


+ + + + + + + +
V
$E = \frac{V}{d}$
d
- - - - - - - -

- a Hall e.m.f. V_{hall} is in the direction perpendicular to the plane containing *i* and *H*.

- The sign of the Hall e.m.f. depends on the sign of the charge carriers

(The range of fields measurable by these devices is typically 0.4 A/m ~ 4 × 10^6 A/m (5 × 10^{-3}~5 × 10^4 Oe). Accuracy of measurement is typically 1 %.)

$$\vec{J} = ne\vec{v}$$

n : the number of charge carries per unit volume

J : the current density (v : velocity)

$$\vec{v} = \frac{\vec{J}}{ne}$$

$$\vec{E}_{Hall} = \mu_0 \vec{v} \times \vec{H}$$

$$= \mu_0 \vec{J} \times \frac{\vec{H}}{ne}$$

$$= \mu_0 R_H \vec{J} \times \vec{H}$$

(by replacing 1/*ne* by the term R_H, which is call the Hall constant)

- The electrical field $E_{Hall}\left(\text{volt}/\text{meter}\right)$ can be expressed by the from $E = \dfrac{V}{l_z}$, where V is the potential difference over a distance l_z

$$\vec{V} = \mu_0 R_H \vec{J} \times \vec{H} l_z$$

– The value of the Hall coefficient R_H is typically 10^{-10} m^3 per coulomb

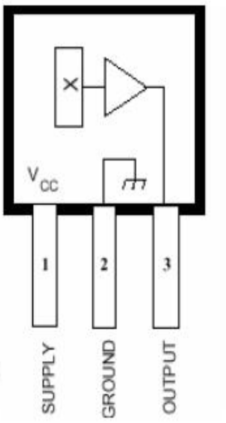
Material	R_H (m^3/C)
Li	-1.7×10^{-10}
In	$+1.59\times10^{-10}$
Sb	-1.98×10^{-9}
Bi	-5.4×10^{-7}

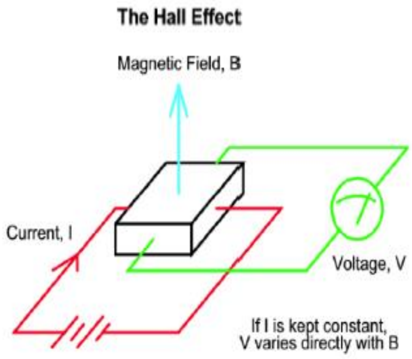
The Hall effect magnetometers can be fabricated with very small active areas down to 10^{-6} m^2 which can therefore be used to measure the magnetic field with high spatial resolution

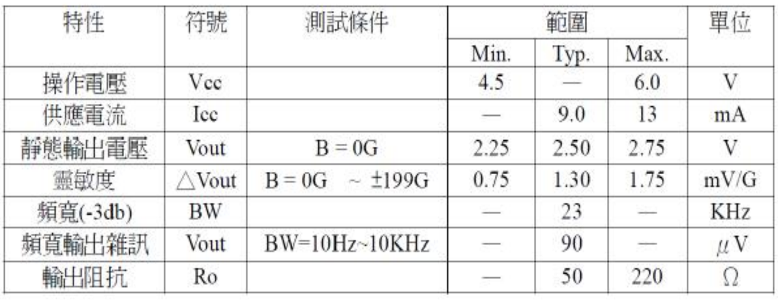
磁性量測

磁性量測原理與類型

霍爾效應高斯計







特性	符號	測試條件	範圍			單位
			Min.	Typ.	Max.	
操作電壓	Vcc		4.5	—	6.0	V
供應電流	Icc		—	9.0	13	mA
靜態輸出電壓	Vout	B = 0G	2.25	2.50	2.75	V
靈敏度	△Vout	B = 0G ~ ±199G	0.75	1.30	1.75	mV/G
頻寬(-3db)	BW		—	23	—	KHz
頻寬輸出雜訊	Vout	BW=10Hz~10KHz	—	90	—	μV
輸出阻抗	Ro		—	50	220	Ω

②Magnetoresistors

(How does the presence of a magnetic field alter the resistance of a material?)

- Magnetoresistance is the change in electrical resistance of a material when subjected to a magnetic field.
- Generally, the resistance increase when a field is applied but is nonlinear.
- In all materials, the effect of magnetic field on resistance is greater when the perpendicular to the direction of current flow.

- In ferromagnetic materials the change in resistance can be quite large. $\Delta R/R$ = 2% at saturation in Ni
  $\Delta R/R$ = 0.3% at saturation in Fe

field measuring device

Bi, the magnetoresistance increase by 150% in a field of 9.5×10^5 A/m (1.2T)

InSb-NiSb undergoes a 300% change in magnetoresistance in a field of 2.3×10^5 A/m (0.3T)

- Unfortunately the effects of temperature on the resistance of this material are also large and this limits its applications.

③Magnetostrictive devices

(Can the change in length of a ferromagnet in the presence of a field be used to measure that field?)

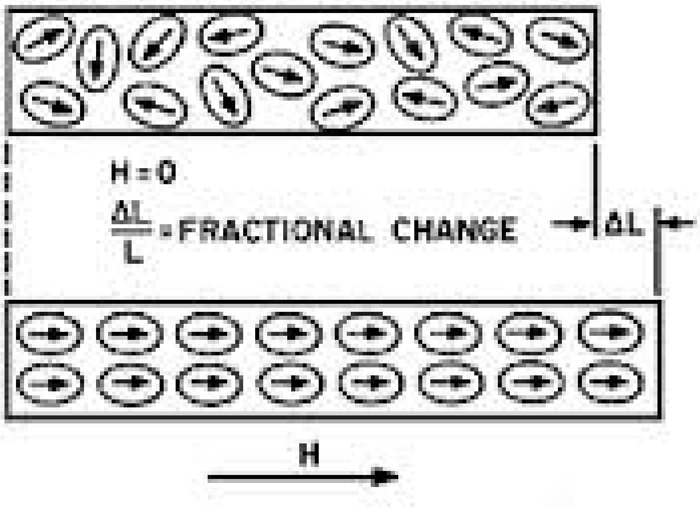
– When a specimen of magnetic material is subjected to a magnetic field changes in shape of the specimen occur. This phenomenon is known as magnetostriction and it is most often demonstrated by measuring the factional change in length *Δl/l* of a specimen as it is magnetized.

- The effect is quite small in most materials but in ferromagnetic it is typically of the order of $\Delta l/l = 10^{-6}$ which is measurable by resistive strain gauge or by optical techniques.

- The magnetostriction and magnetoresistance are closely related, both being generated by the spin orbit coupling so that the electron distribution at each ionic site is rotated.

- This change in electron distribution alters the scattering undergone by the conduction electrons (magnetoresistance)

The rotation of the moments also leads to a change in the ionic spacing (magnetorestriction)



- other methods:
  1. Torque magnetometers
  2. Susceptibility balances
  3. SQUID

1. Torque magnetometers

(How can the magnetic moment or magnetization be found from the torque exerted by a known external field?)

- The torque magnetometer is used mainly for anisotropy measurement on short specimens. It is based on the fact that a magnetic dipole m in an external magnetic field H in free space experiences a torque τ.

$$\vec{\tau} = \mu_0 \vec{m} \times \vec{H}$$

- The specimen is aligned so that its magnetization lies in the plane of rotation of the field or the sample is rotated in the plane defined by the magnetic field H and the magnetization M.

- The angle ϕ through which the torsion fiber is twisted is dependent on the length of the fiber and its shear modulus as well as the torque.

- ϕ is proportional to the turning force on the specimen

$$\phi = \text{constant} \times \tau$$

$$= \text{constant} \times \mu_0 m \times H$$

$$\left(\begin{array}{l}\text{If is the angle between } m \text{ and H}\\ \phi = \text{constant} \times \mu_0 m H \sin\alpha\end{array}\right)$$

$$\left.\begin{array}{l}\left.\begin{array}{l}\mu_0\\ H\\ \text{constant}\end{array}\right\}\ \text{are known}\\ \qquad \alpha \quad \text{can be measured}\end{array}\right\} \longrightarrow \begin{array}{l}\text{the magnetic moment}(m)\\ \text{can be calculated}\end{array}$$

2.Susceptibility balances

(How can the magnetization be found from the force exerted on a specimen by a known field?)

- Two types of force-balance methods have been devised for determining the magnetization M or equivalently the susceptibility χ

analytical balance
torsion balance

$\left.\begin{matrix}\text{analytical balance}\\ \text{torsion balance}\end{matrix}\right\}$ depend on measuring the linear force on a sample suspended in a magnetic field gradient

- The specimen is suspended using a long string in a magnetic field with a constant field gradient
- The force on the specimen of volume (V) and magnetization (M) is:

$$\boxed{\begin{aligned} F_x &= \mu_0 m \frac{dH}{d\chi} \\ &= -\mu_0 VM \frac{dH}{d\chi} \end{aligned}} \qquad (M = \frac{m}{V})$$

$$\because \chi = {}^{M}\!/\!{}_{H}$$

$$\therefore F_x = -\mu_0 VxH \frac{dH}{d\chi}$$

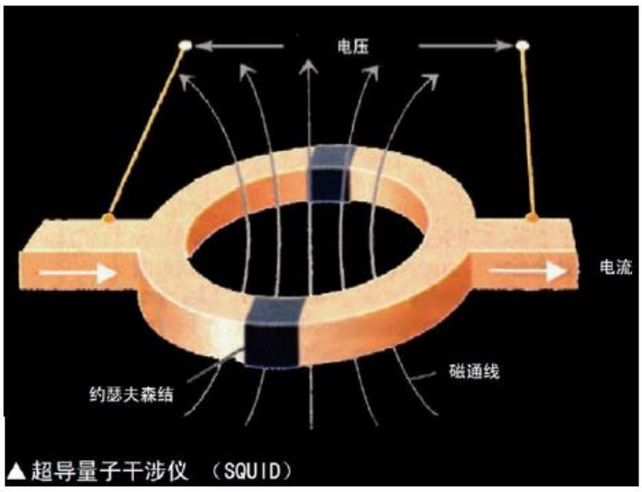
- The force on the specimen is proportional to its susceptibility.

磁性量測

磁性量測原理與類型

超導量子干涉儀

超導量子干涉儀(SQUID)，為利用Josephson效應製備的最精確的磁場探測器之一。最高精度達到5×10^{-18} T。SQUID高精度的磁測量能夠檢測出地球磁場的幾億分之一的變化，或是探測10^{-9} T 到10^{-6} T間的生物磁場。



▲超导量子干涉仪 （SQUID）